Hash Tables

CS 121: Data Structures

START RECORDING

Outline

- Attendance quiz
- Review: Comparable and Equals from "Symbol Tables" slides
- Hash functions
- Separate chaining
- Linear probing
- Context

Attendance Quiz

Attendance Quiz: Symbol Tables

- Scan the QR code, or find today's attendance quiz under the "Quizzes" tab on Canvas
- Password: to be announced



Key	Value				
С	3				
Α	1				
Z	26				
В	2				

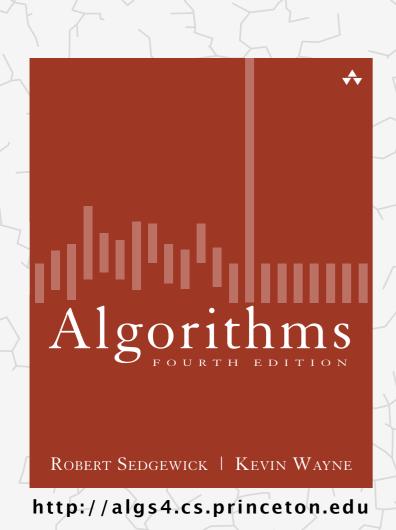
Attendance Quiz: Symbol Tables

- Write your name and the date
- Draw the data structures of symbol tables containing the data shown at the right for:
 - A symbol table implemented using an unordered linked list
 - A symbol table implemented using ordered arrays

Key	Value				
С	3				
Α	1				
Z	26				
В	2				

No Office Hours This Friday (3/31)

Algorithms



context

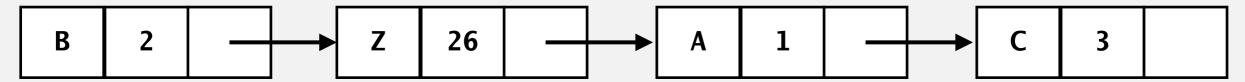
3.4 HASH TABLES

- hash functions
- separate chaining
- linear probing

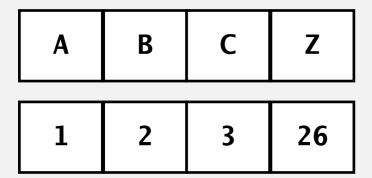
Symbol table implementations: summary

implementation	guarantee			average case			ordered	key
	search	insert	delete	search hit	insert	delete	ops?	interface
sequential search (unordered list)	N	N	N	½ N	N	½ N		equals()
binary search (ordered array)	lg N	N	N	lg N	½ N	½ N	•	compareTo()

Unordered List



Ordered Arrays



- Q. Can we do better?
- A. Yes, but with different access to the data.

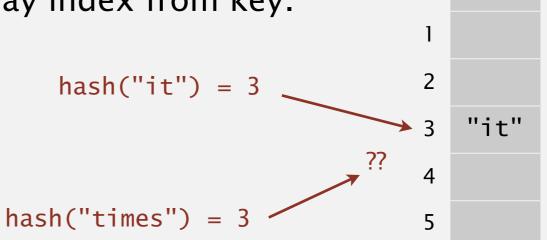
Symbol table implementations: this lecture

implementation	guarantee			average case			ordered	key
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sequential search (unordered list)	N	N	N	½ N	N	½ N		equals()
binary search (ordered array)	lg N	N	N	lg N	½ N	½ N	~	compareTo()
This lecture	N	N	N	3-5 *	3-5 *	3-5 *		equals() hashCode()

Hashing: basic plan

Save items in a key-indexed table (index is a function of the key).

Hash function. Method for computing array index from key.



0

Issues.

- Computing the hash function.
- · Equality test: Method for checking whether two keys are equal.
- Collision resolution: Algorithm and data structure to handle two keys that hash to the same array index.

3.4 HASH TABLES

- hash functions
- separate chaining
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Algorithms

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Computing the hash function

Idealistic goal. Scramble the keys uniformly to produce a table index.

- Efficiently computable.
- Each table index equally likely for each key.

thoroughly researched problem, still problematic in practical applications

key table

index

Ex 1. Phone numbers.

- Bad: first three digits.
- Better: last three digits.

Ex 2. Social Security numbers.

- Bad: first three digits.

 573 = California, 574 = Alaska (assigned in chronological order within geographic region)
- Better: last three digits.

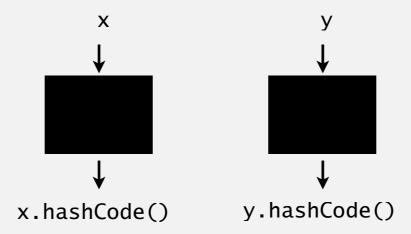
Practical challenge. Need different approach for each key type.

Java's hash code conventions

All Java classes inherit a method hashCode(), which returns a 32-bit int.

Requirement. If x.equals(y), then (x.hashCode() == y.hashCode()).

Highly desirable. If !x.equals(y), then (x.hashCode() != y.hashCode()).



Default implementation. Memory address of x.

Legal (but poor) implementation. Always return 17.

Customized implementations. Integer, Double, String, File, URL, Date, ...

User-defined types. Users are on their own.

Implementing hash code: integers, booleans, and doubles

Java library implementations

```
public final class Integer
{
   private final int value;
   ...

public int hashCode()
   { return value; }
}
```

```
public final class Double
{
   private final double value;
   ...

public int hashCode()
   {
    long bits = doubleToLongBits(value);
    return (int) (bits ^ (bits >>> 32));
   }
}
```

convert to IEEE 64-bit representation; xor most significant 32-bits with least significant 32-bits

Warning: -0.0 and +0.0 have different hash codes

Implementing hash code: strings

Java library implementation

```
public final class String
{
    private final char[] s;
    ...

public int hashCode()
    {
        int hash = 0;
        for (int i = 0; i < length(); i++)
            hash = s[i] + (31 * hash);
        return hash;
    }
    ith character of s
}</pre>
```

char	Unicode				
'a'	97				
'b'	98				
'c'	99				

- Horner's method to hash string of length L: L multiplies/adds.
- Equivalent to $h = s[0] \cdot 31^{L-1} + ... + s[L-3] \cdot 31^2 + s[L-2] \cdot 31^1 + s[L-1] \cdot 31^0$.

```
Ex. String s = \text{"call"};

int code = s.hashCode(); \longleftrightarrow 3045982 = 99.31^3 + 97.31^2 + 108.31^1 + 108.31^0

= 108 + 31 \cdot (108 + 31 \cdot (97 + 31 \cdot (99)))

(Horner's method)
```

Implementing hash code: strings

Performance optimization.

- Cache the hash value in an instance variable.
- Return cached value.

```
public final class String
                                                        cache of hash code
   private int hash = 0;
   private final char[] s;
   public int hashCode()
      int h = hash;
                                                        return cached value
      if (h != 0) return h;
      for (int i = 0; i < length(); i++)
         h = s[i] + (31 * h);
      hash = h;
                                                        store cache of hash code
      return h;
   }
```

Q. What if hashCode() of string is 0?

Implementing hash code: user-defined types

```
public final class Transaction implements Comparable<Transaction>
   private final String who;
   private final Date
                          when;
   private final double amount;
   public Transaction(String who, Date when, double amount)
   { /* as before */ }
   public boolean equals(Object y)
   { /* as before */ }
   public int hashCode()
                                  nonzero constant
      int hash = 17; *
                                                                           for reference types,
                                                                          use hashCode()
      hash = 31*hash + who.hashCode();
      hash = 31*hash + when.hashCode();
                                                                          for primitive types,
      hash = 31*hash + ((Double) amount).hashCode();
                                                                          use hashCode()
      return hash;
                                                                          of wrapper type
                        typically a small prime
```

Hash code design

"Standard" recipe for user-defined types.

- Combine each significant field using the 31x + y rule.
- If field is a primitive type, use wrapper type hashCode().
- If field is null, return 0.
- If field is a reference type, use hashCode(). ← applies rule recursively
- If field is an array, apply to each entry. ← or use Arrays.deepHashCode()

In practice. Recipe works reasonably well; used in Java libraries. In theory. Keys are bitstring; "universal" hash functions exist.

Basic rule. Need to use the whole key to compute hash code; consult an expert for state-of-the-art hash codes.

Modular hashing

Hash code. An int between -2³¹ and 2³¹ - 1.

Hash function. An int between 0 and M - 1 (for use as array index).

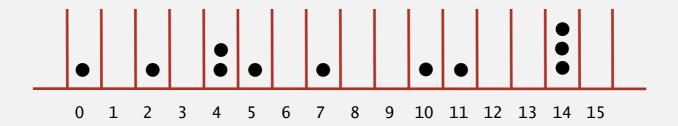
typically a prime or power of 2

```
X
 private int hash(Key key)
 { return key.hashCode() % M; }
bug
                                                                x.hashCode()
 private int hash(Key key)
    return Math.abs(key.hashCode()) % M; }
1-in-a-billion bug
                                                                   hash(x)
                     hashCode() of "polygenelubricants" is -231
 private int hash(Key key)
 { return (key.hashCode() & 0x7fffffff) % M; }
correct
```

Uniform hashing assumption

Uniform hashing assumption. Each key is equally likely to hash to an integer between 0 and M-1.

Bins and balls. Throw balls uniformly at random into *M* bins.



Birthday problem. Expect two balls in the same bin after $\sim \sqrt{\pi M/2}$ tosses.

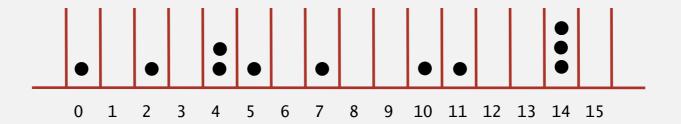
Coupon collector. Expect every bin has ≥ 1 ball after $\sim M \ln M$ tosses.

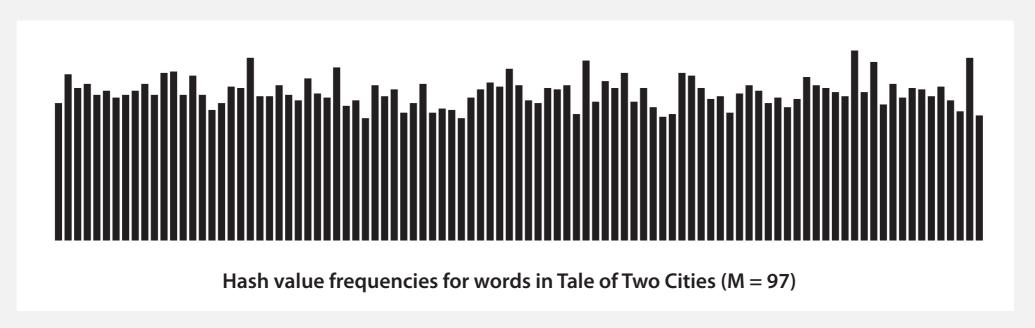
Load balancing. After M tosses, expect most loaded bin has Θ ($\log M / \log \log M$) balls.

Uniform hashing assumption

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Bins and balls. Throw balls uniformly at random into *M* bins.





Java's String data uniformly distribute the keys of Tale of Two Cities

3.4 HASH TABLES

- hash functions
- separate chaining
- linear probing
- context

Algorithms

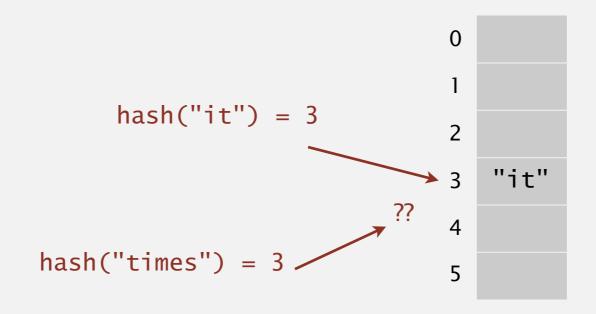
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Collisions

Collision. Two distinct keys hashing to same index.

- Birthday problem ⇒ can't avoid collisions unless you have a ridiculous (quadratic) amount of memory.
- Coupon collector + load balancing \Rightarrow collisions are evenly distributed.

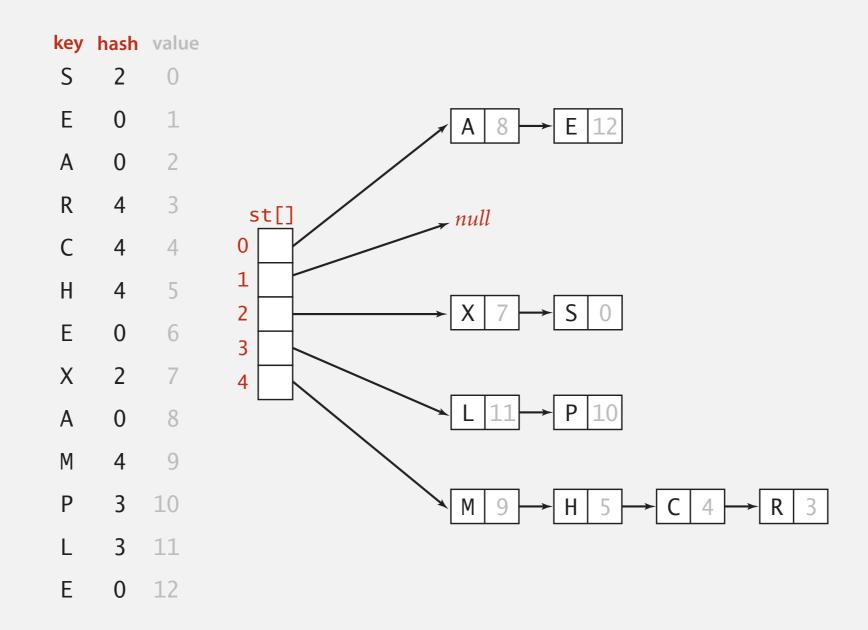


Challenge. Deal with collisions efficiently.

Separate-chaining symbol table

Use an array of M < N linked lists. [H. P. Luhn, IBM 1953]

- Hash: map key to integer i between 0 and M-1.
- Insert: put at front of i^{th} chain (if not already there).
- Search: need to search only *i*th chain.



Separate-chaining symbol table: Java implementation

```
public class SeparateChainingHashST<Key, Value>
  private int M = 97;
                     // number of chains
  private Node[] st = new Node[M]; // array of chains
  private static class Node
     private Object key; ← no generic array creation
     private Node next;
  private int hash(Key key)
  { return (key.hashCode() & 0x7fffffff) % M; }
  public Value get(Key key) {
     int i = hash(key);
     for (Node x = st[i]; x != null; x = x.next)
       if (key.equals(x.key)) return (Value) x.val;
     return null;
```

array doubling and halving code omitted

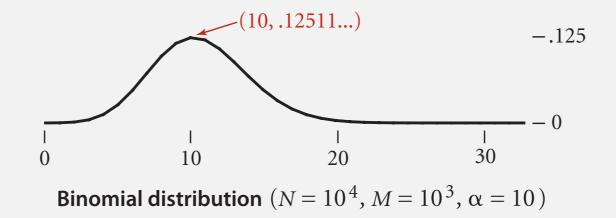
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public class SeparateChainingHashST<Key, Value>
                       // number of chains
  private int M = 97;
  private Node[] st = new Node[M]; // array of chains
  private static class Node
     private Object key;
     private Object val;
     private Node next;
  private int hash(Key key)
  { return (key.hashCode() & 0x7fffffff) % M; }
  public void put(Key key, Value val) {
     int i = hash(key);
     for (Node x = st[i]; x != null; x = x.next)
        if (key.equals(x.key)) { x.val = val; return; }
     st[i] = new Node(key, val, st[i]);
```

Analysis of separate chaining

Proposition. Under uniform hashing assumption, prob. that the number of keys in a list is within a constant factor of N/M is extremely close to 1.

Pf sketch. Distribution of list size obeys a binomial distribution.



equals() and hashCode()

Consequence. Number of probes for search/insert is proportional to N/M.

- M too large \Rightarrow too many empty chains.
- M too small \Rightarrow chains too long.
- Typical choice: $M \sim N/4 \Rightarrow$ constant-time ops.

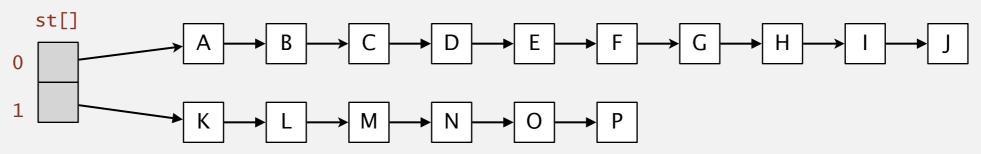
M times faster than sequential search

Resizing in a separate-chaining hash table

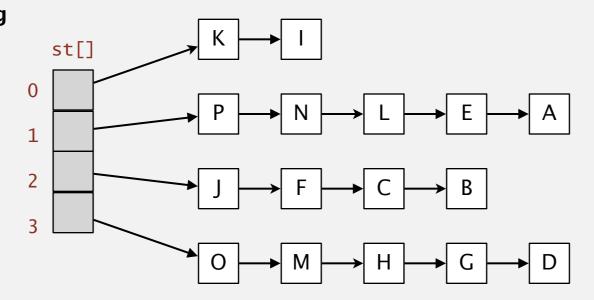
Goal. Average length of list N/M = constant.

- Double size of array M when $N/M \ge 8$.
- Halve size of array M when $N/M \le 2$.
- Need to rehash all keys when resizing. ← x.hashCode() does not change but hash(x) can change

before resizing



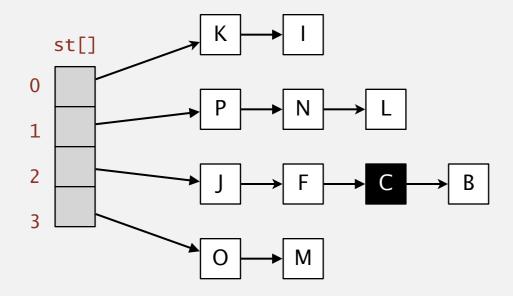
after resizing



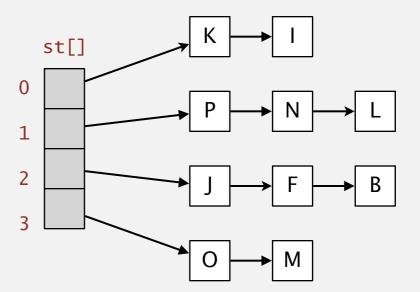
Deletion in a separate-chaining hash table

- Q. How to delete a key (and its associated value)?
- A. Easy: need only consider chain containing key.

before deleting C



after deleting C



Symbol table implementations: summary

implementation -	guarantee			average case			ordered	key
	search	insert	delete	search hit	insert	delete	ops?	interface
sequential search (unordered list)	N	N	N	½ N	N	½ N		equals()
binary search (ordered array)	lg N	N	N	lg N	½ N	½ N	✓	compareTo()
separate chaining	N	N	N	3-5 *	3-5 *	3-5 *		equals() hashCode()

^{*} under uniform hashing assumption

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Algorithms

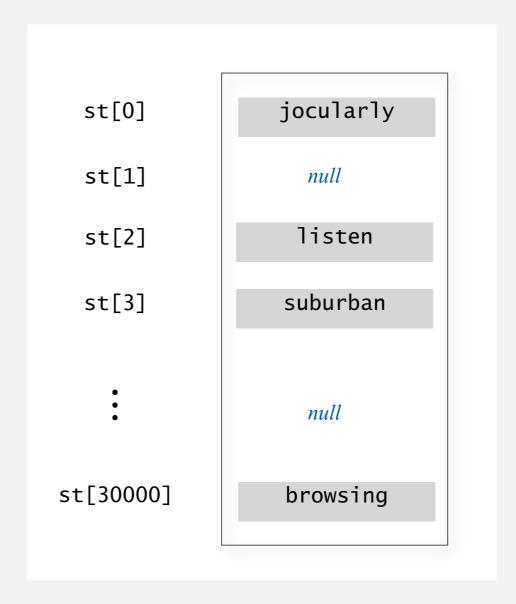
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Collision resolution: open addressing

Open addressing. [Amdahl-Boehme-Rocherster-Samuel, IBM 1953]

When a new key collides, find next empty slot, and put it there.



linear probing (M = 30001, N = 15000)

Linear Probing Demo

Linear-probing hash table summary

Hash. Map key to integer i between 0 and M-1.

Insert. Put at table index i if free; if not try i+1, i+2, etc.

Search. Search table index i; if occupied but no match, try i+1, i+2, etc.

Note. Array size M must be greater than number of key-value pairs N.



M = 16

Linear-probing symbol table: Java implementation

```
public class LinearProbingHashST<Key, Value>
  private int M = 30001;
  private Value[] vals = (Value[]) new Object[M];
  private Key[] keys = (Key[]) new Object[M];
  private int hash(Key key)
  { return (key.hashCode() & 0x7fffffff) % M; }
  private void put(Key key, Value val) { /* next slide */ }
  public Value get(Key key)
     for (int i = hash(key); keys[i] != null; i = (i+1) % M)
        if (key.equals(keys[i]))
            return vals[i];
     return null;
   }
```

array doubling and halving code omitted

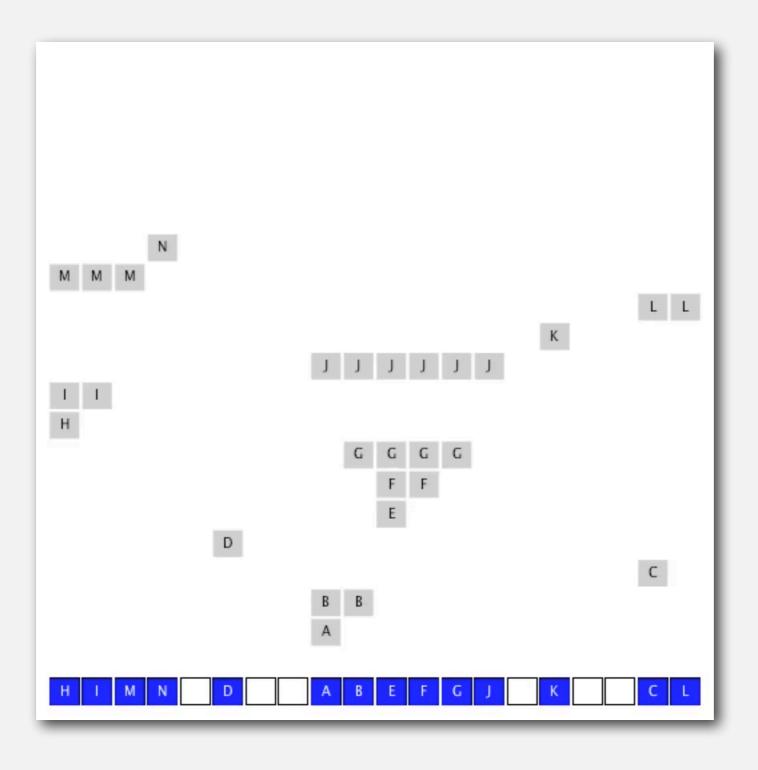
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   private int M = 30001;
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   private Key[] keys = (Key[]) new Object[M];
   private int hash(Key key) { /* previous slide */ }
   private Value get(Key key) { /* previous slide */ }
   public void put(Key key, Value val)
     int i;
      for (i = hash(key); keys[i] != null; i = (i+1) % M)
        if (keys[i].equals(key))
             break;
      keys[i] = key;
     vals[i] = val;
```

Clustering

Cluster. A contiguous block of items.

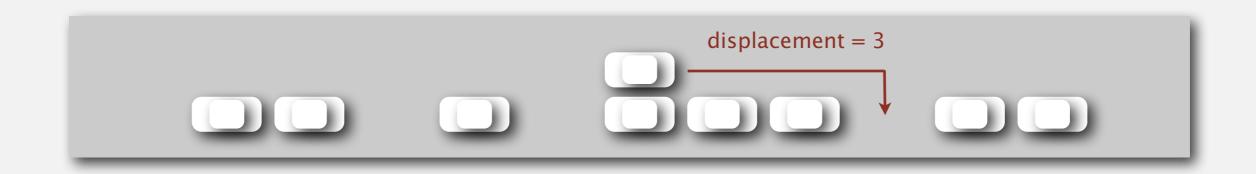
Observation. New keys likely to hash into middle of big clusters.



Knuth's parking problem

Model. Cars arrive at one-way street with M parking spaces. Each desires a random space i: if space i is taken, try i+1, i+2, etc.

Q. What is mean displacement of a car?



Half-full. With M/2 cars, mean displacement is $\sim 3/2$.

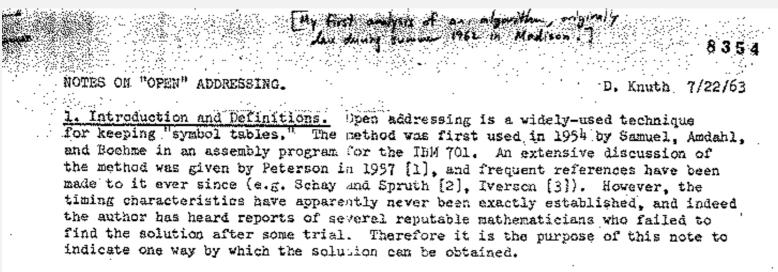
Full. With M cars, mean displacement is $\sim \sqrt{\pi M/8}$.

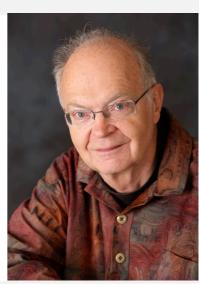
Analysis of linear probing

Proposition. Under uniform hashing assumption, the average # of probes in a linear probing hash table of size M that contains $N = \alpha M$ keys is:

$$\sim \frac{1}{2} \left(1 + \frac{1}{1 - \alpha} \right) \qquad \sim \frac{1}{2} \left(1 + \frac{1}{(1 - \alpha)^2} \right)$$
 search hit search miss / insert

Pf.





Parameters.

- M too large \Rightarrow too many empty array entries.
- M too small \Rightarrow search time blows up.
- Typical choice: $\alpha = N/M \sim \frac{1}{2}$. # probes for search hit is about 3/2 # probes for search miss is about 5/2

Resizing in a linear-probing hash table

Goal. Average length of list $N/M \leq \frac{1}{2}$.

- Double size of array M when $N/M \ge \frac{1}{2}$.
- Halve size of array M when $N/M \le \frac{1}{8}$.
- Need to rehash all keys when resizing.

before resizing

	0	1	2	3	4	5	6	7
keys[]		E	S			R	Α	
vals[]		1	0			3	2	

after resizing

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
keys[]					Α		S				E				R	
vals[]					2		0				1				3	

Deletion in a linear-probing hash table

- Q. How to delete a key (and its associated value)?
- A. Requires some care: can't just delete array entries.

before deleting S

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
keys[]	Р	М			Α	С	S	Н	L		E				R	X
vals[]	10	9			8	4	0	5	11		12				3	7



ST implementations: summary

implementation		guarantee			average case	ordered	key	
implementation	search	insert	delete	search hit	insert	delete	ops?	interface
sequential search (unordered list)	N	N	N	½ N	N	½ N		equals()
binary search (ordered array)	lg N	N	N	lg N	½ N	½ N	•	compareTo()
separate chaining	N	N	N	3-5 *	3-5 *	3-5 *		equals() hashCode()
linear probing	N	N	N	3-5 *	3-5 *	3-5 *		equals() hashCode()

^{*} under uniform hashing assumption

START RECORDING

Attendance Quiz

Attendance Quiz: Hashing

- Write your name and the date
- Describe the relationship between "hash codes" and "hash functions"
- What properties should hash codes have?
- What properties should hash functions have?

Algorithms

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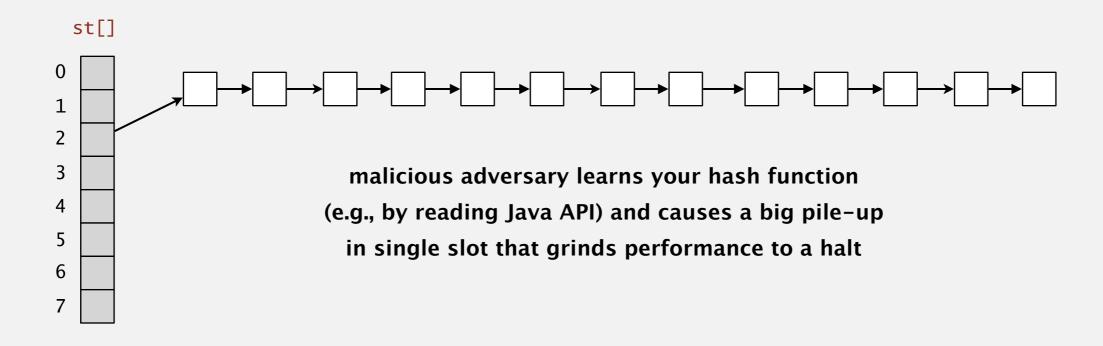
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3.4 HASH TABLES

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- linear probing
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War story: algorithmic complexity attacks

- Q. Is the uniform hashing assumption important in practice?
- A. Obvious situations: aircraft control, nuclear reactor, pacemaker.
- A. Surprising situations: denial-of-service attacks.



Real-world exploits. [Crosby-Wallach 2003]

- IDS server: send carefully chosen packets to DOS the server, using less bandwidth than a dial-up modem.
- Perl 5.8.0: insert carefully chosen strings into associative array.
- Linux 2.4.20 kernel: save files with carefully chosen names.

War story: String hashing in Java

String hashCode() in Java 1.1.

- For long strings: only examine 8-9 evenly spaced characters.
- Benefit: saves time in performing arithmetic.

```
public int hashCode()
{
   int hash = 0;
   int skip = Math.max(1, length() / 8);
   for (int i = 0; i < length(); i += skip)
      hash = s[i] + (37 * hash);
   return hash;
}</pre>
```

Downside: great potential for bad collision patterns.

War story: algorithmic complexity attacks

A Java bug report.

Jan Lieskovsky 2011-11-01 10:13:47 EDT

Description

Julian Wälde and Alexander Klink reported that the String.hashCode() hash function is not sufficiently collision resistant. hashCode() value is used in the implementations of HashMap and Hashtable classes:

http://docs.oracle.com/javase/6/docs/api/java/util/HashMap.html http://docs.oracle.com/javase/6/docs/api/java/util/Hashtable.html

A specially-crafted set of keys could trigger hash function collisions, which can degrade performance of HashMap or Hashtable by changing hash table operations complexity from an expected/average O(1) to the worst case O(n). Reporters were able to find colliding strings efficiently using equivalent substrings and meet in the middle techniques.

This problem can be used to start a denial of service attack against Java applications that use untrusted inputs as HashMap or Hashtable keys. An example of such application is web application server (such as tomcat, see bug #750521) that may fill hash tables with data from HTTP request (such as GET or POST parameters). A remote attack could use that to make JVM use excessive amount of CPU time by sending a POST request with large amount of parameters which hash to the same value.

This problem is similar to the issue that was previously reported for and fixed in e.g. perl:

http://www.cs.rice.edu/~scrosby/hash/CrosbyWallach_UsenixSec2003.pdf

Algorithmic complexity attack on Java

Goal. Find family of strings with the same hash code.

Solution. The base-31 hash code is part of Java's string API.

key	hashCode()
"Aa"	2112
"BB"	2112

key	hashCode()
"AaAaAaAa"	-540425984
"AaAaAaBB"	-540425984
"AaAaBBAa"	-540425984
"AaAaBBBB"	-540425984
"AaBBAaAa"	-540425984
"AaBBAaBB"	-540425984
"AaBBBBAa"	-540425984
"AaBBBBBB"	-540425984

key	hashCode()
"BBAaAaAa"	-540425984
"BBAaAaBB"	-540425984
"BBAaBBAa"	-540425984
"BBAaBBBB"	-540425984
"BBBBAaAa"	-540425984
"BBBBAaBB"	-540425984
"BBBBBBAa"	-540425984
"BBBBBBBB"	-540425984

2N strings of length 2N that hash to same value!

Diversion: one-way hash functions

One-way hash function. "Hard" to find a key that will hash to a desired value (or two keys that hash to same value).

```
Ex. MD4, MD5, SHA-0, SHA-1, SHA-2, WHIRLPOOL, RIPEMD-160, ....
```

```
String password = args[0];
MessageDigest sha1 = MessageDigest.getInstance("SHA1");
byte[] bytes = sha1.digest(password);

/* prints bytes as hex string, such as:
   * B5764C27DB3299C4DC18ED922890F90EA6C336F1
   */
```

Applications. Digital fingerprint, message digest, storing passwords. Caveat. Too expensive for use in ST implementations.

Google fixes nightmare Android bug that stopped user from calling 911

An integer overflow/underflow crash lets misbehaving apps lock users out of 911.

Ron Amadeo - 1/5/2022, 6:09 PM

Android's January security patch is out, and it's addressing one of the nastiest Android bugs to come up in some time: certain apps can stop you from contacting 911 or other worldwide emergency services numbers.

In early December, a harrowing tale popped up in the GooglePixel subreddit from a user whose Pixel 3 crashed when they needed it most: while dialing 911 for their grandmother who "appeared to be having a stroke." The whole phone subsystem seemed to immediately crash upon calling emergency services, with user "KitchenPicture5849" saying they couldn't get the call to connect or hang up to try the call again. Luckily, a nearby landline was available after their Android phone let them down, and emergency services was able to be contacted.

Creative Use of Hash Codes Can Lead to Bugs

- Android allows different apps to be used as dialers (e.g., the default dialer, Skype, Microsoft Teams, etc.).
- To determine which dialer to use for a call, the dialers are sorted. As a fallback, when all other attributes are the same, hash codes were compared to give an arbitrary but consistent ordering.

The buggy code:

```
return account1.hashCode() - account2.hashCode();
```

Fixed code:

```
return Integer.compare(account1.hashCode(), account2.hashCode());
```

- Why would this cause undefined behavior (e.g., "crashing")? Consider:
 - account1.hashCode() → MAX_INTEGER
 - account2.hashCode() → -20
 - account3_hashCode() → 30

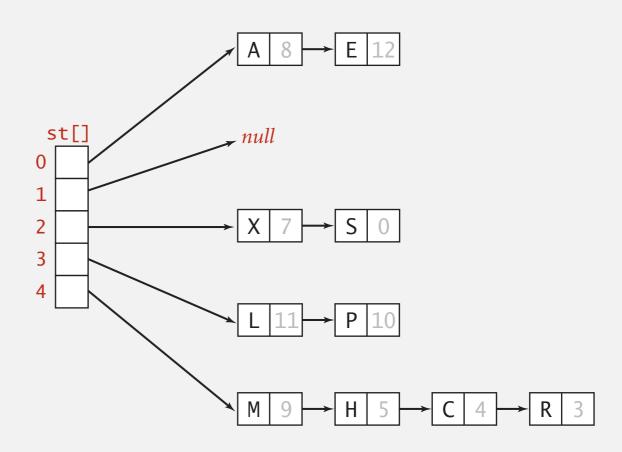
Separate chaining vs. linear probing

Separate chaining.

- Performance degrades gracefully.
- Clustering less sensitive to poorly-designed hash function.

Linear probing.

- Less wasted space.
- Better cache performance.



0	- 1	2	3	4	5	6	/	8	9	10	11	12	13	14	15
Р	М			А	С	S	Н	L		Е				R	Х
10	9			8	4	0	5	11		12				3	7

keys[]
vals[]

Hashing: variations on the theme

Many improved versions have been studied.

Two-probe hashing. [separate-chaining variant]

- Hash to two positions, insert key in shorter of the two chains.
- Reduces expected length of the longest chain to $\log \log N$.

Double hashing. [linear-probing variant]

- Use linear probing, but skip a variable amount, not just 1 each time.
- Effectively eliminates clustering.
- Can allow table to become nearly full.
- More difficult to implement delete.

Cuckoo hashing. [linear-probing variant]

- Hash key to two positions; insert key into either position; if occupied, reinsert displaced key into its alternative position (and recur).
- Constant worst-case time for search.

Hash tables vs. balanced search trees

Hash tables.

- Simpler to code.
- No effective alternative for unordered keys.
- Faster for simple keys (a few arithmetic ops versus $\log N$ compares).
- Better system support in Java for strings (e.g., cached hash code).

Balanced search trees.

- Stronger performance guarantee.
- Support for ordered ST operations.
- Easier to implement compareTo() correctly than equals() and hashCode().

Java system includes both.

- Red-black BSTs: java.util.TreeMap, java.util.TreeSet.
- Hash tables: java.util.HashMap, java.util.IdentityHashMap.