CS140:

Backgrounder on Number Systems and Binary Numbers

John Magee

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Some slides credit: Aaron Stevens

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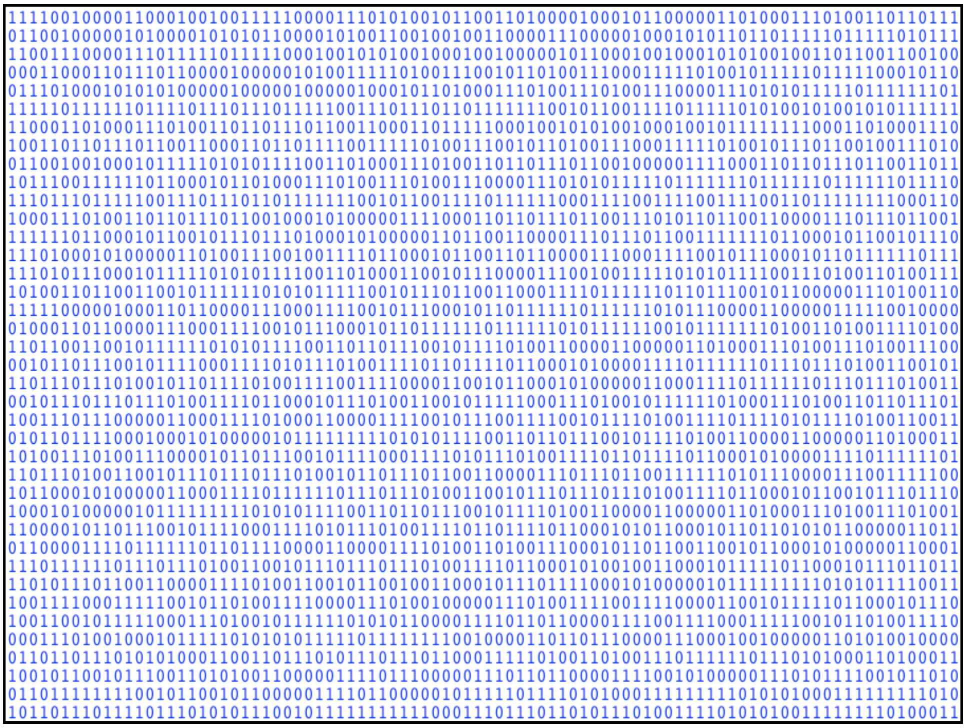

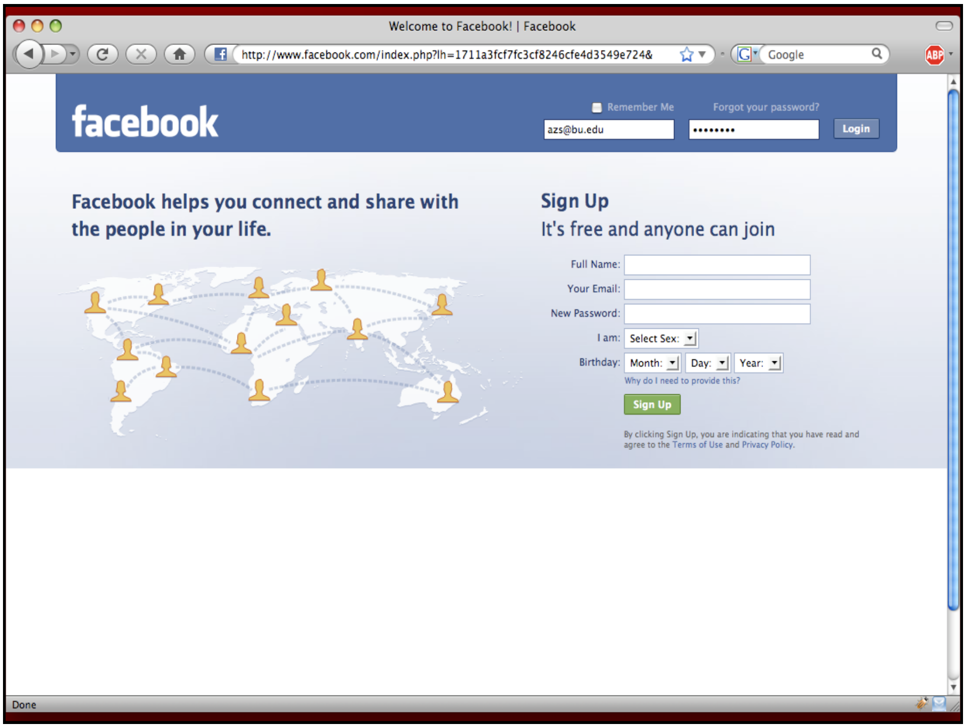

!!! MATH WARNING !!!

TODAY'S LECTURE CONTAINS TRACE AMOUNTS OF
ARITHMETIC AND ALGEBRA

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Overview/Questions

– What gives a number its value?
– What is a number system?
– I've heard that computers use binary numbers. What's a binary number?
– What kind of numbers do computers store and manipulate?

Backgrounder: Numbers

Natural Numbers
Zero and any number obtained by repeatedly adding one to it.

Examples: 100, 0, 45645, 32

Negative Numbers
A value less than 0, with a – sign

Examples: -24, -1, -45645, -32

Numbers

Integers
A natural number, a negative number, zero

Examples: 249, 0, -45645, -32

Rational Numbers
An integer or the quotient of two integers

Examples: -249, -1, 0, 3/7, -2/5

Numbering Systems

A numbering system assigns meaning to the position of the numeric symbols.

For example, consider this set of symbols:

642

What number is it? Why?

Numbering Systems

It depends on the numbering system.

642 is 600 + 40 + 2 in **BASE 10**

The **base** of a number determines the number of digits (e.g. symbols) and the value of digit positions

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Positional Notation

Continuing with our example…

642 in base 10 *positional notation* is:

$$\begin{aligned} & 6 \times 10^2 = 6 \times 100 = 600 \\ + & 4 \times 10^1 = 4 \times 10 = 40 \\ + & 2 \times 10^0 = 2 \times 1 = 2 \quad = 642 \text{ in base } 10 \end{aligned}$$

This number is in base 10

The power indicates the position of the number

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Positional Notation

642 is $6_3 * 10^2 + 4_2 * 10^1 + 2_1 * 10^0$

As a general form:

$$d_n * B^{n-1} + d_{n-1} * B^{n-2} + \ldots + d_1 * B^0$$

B is the base of the number

n is the number of digits in the number

d is the digit in the i^{th} position in the number

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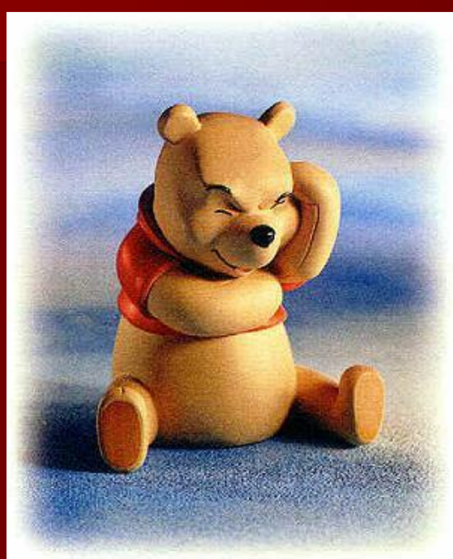
What Would Pooh Do?



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Binary Numbers

Digital computers are made up of electronic circuits, which have exactly 2 states: on and off.

Computers use a numbering system which has exactly 2 symbols, representing on and off.

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Binary Numbers

Decimal is base 10 and has 10 digits:

0,1,2,3,4,5,6,7,8,9

Binary is base 2 and has 2, so we use only 2 symbols:

0,1

For a given base, valid numbers will only contain the digits in that base, which range from 0 up to (but not including) the base.

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Counting…

Let's remember Kindergarten…

Binary Numbers and Computers

A binary digit or bit can take on only these two values.

Low Voltage = 0	
High Voltage = 1	all bits have 0 or 1

Binary numbers are built by concatenating a string of bits together.
Example: 10101010

Positional Notation: Binary Numbers

Recall this general form:

$$d_n * B^{n-1} + d_{n-1} * B^{n-2} + ... + d_1 * B^0$$

The same can be applied to base-2 numbers:

1011(binary) =	$1 * 2^3$	(1 * 8)
	$+ 0 * 2^2$	(0 * 4)
	$+ 1 * 2^1$	(1 * 2)
	$+ 1 * 2^0$	(1 * 1)

1011(binary) = 8 + 0 + 2 + 1 = 11(decimal)

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Converting Binary to Decimal

What is the decimal equivalent of the binary number 01101110?

(you try it! Work left-to-right)

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Converting Binary to Decimal

What is the decimal equivalent of the binary number 01101110?

$0 \times 2^7 = 0 \times 128 = 0$
$+ 1 \times 2^6 = 1 \times 64 = 64$
$+ 1 \times 2^5 = 1 \times 32 = 32$
$+ 0 \times 2^4 = 0 \times 16 = 0$
$+ 1 \times 2^3 = 1 \times 8 = 8$
$+ 1 \times 2^2 = 1 \times 4 = 4$
$+ 1 \times 2^1 = 1 \times 2 = 2$
$+ 0 \times 2^0 = 0 \times 1 = 0$
$= 110$ (decimal)

Converting Binary to Decimal

Try another one. What is the decimal equivalent of the binary number 10101010?

(you try it! Work left-to-right)

Converting Binary to Decimal

Try another one. What is the decimal equivalent of the binary number 10101010?

$1 \times 2^7 = 1 \times 128 = 128$
$+ 0 \times 2^6 = 0 \times 64 = 0$
$+ 1 \times 2^5 = 1 \times 32 = 32$
$+ 0 \times 2^4 = 0 \times 16 = 0$
$+ 1 \times 2^3 = 1 \times 8 = 8$
$+ 0 \times 2^2 = 0 \times 4 = 0$
$+ 1 \times 2^1 = 1 \times 2 = 2$
$+ 0 \times 2^0 = 0 \times 1 = 0$
= 170 (decimal)

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Converting from Decimal to Other Bases

Algorithm (process) for converting number in base 10 to other bases

While (the quotient is not zero)
  Divide the decimal number by the new base*
  Make the remainder the next digit to the left in the answer
  Replace the original decimal number with the quotient

** Using whole number (integer) division only.*
Example: 3 / 2 gives us a quotient of 1 and a remainder 1

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Converting Decimal to Binary

What is the binary equivalent of the decimal number 103?

103 / 2 = 51, remainder 1 → rightmost bit
51 / 2 = 25, remainder 1
25 / 2 = 12, remainder 1
12 / 2 = 6, remainder 0
6 / 2 = 3, remainder 0
3 / 2 = 1, remainder 1
1 / 2 = 0, remainder 1→ leftmost bit

103 (decimal) = 1 1 0 0 1 1 1 (binary)

Converting Decimal to Binary

Now you try one. What is the binary equivalent of the decimal number 201?

Recall the algorithm:

While (the quotient is not zero)
  Divide the decimal number by the new base*
  Make the remainder the next digit to the left in the answer
  Replace the original decimal number with the quotient

Converting Decimal to Binary

What is the binary equivalent of the decimal number 201?

201 / 2 = 100, remainder 1 → rightmost bit
100 / 2 = 50, remainder 0
50 / 2 = 25, remainder 0
25 / 2 = 12, remainder 1
12 / 2 = 6, remainder 0
6 / 2 = 3, remainder 0
3 / 2 = 1, remainder 1
1 / 2 = 0, remainder 1→ leftmost bit

201 (decimal) = 1 1 0 0 1 0 0 1 (binary)

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Binary and Computers

Byte
8 bits – a common unit of computer memory.

Word
A computer word is a group of bits which are passed around together during computation.
The word length of the computer's processor is how many bits are grouped together.

- 8-bit machine (e.g. Nintendo Gameboy, 1989)
- 16-bit machine (e.g. Sega Genesis, 1989)
- 32-bit machines (e.g. Sony PlayStation, 1994)
- 64-bit machines (e.g. Nintendo 64, 1996)

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Common Number Systems

Binary – base 2, has 2 symbols:
0,1

Octal – base 8, has 8 symbols:
0,1,2,3,4,5,6,7

Decimal – base 10, has 10 symbols:
0,1,2,3,4,5,6,7,8,9

Hexadecimal - base 16 has 16 digits:
0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Why Hexadecimal?

Base 16 is a multiple of Base 2:

$16 = 2^4$

Each four bits map to a hex digit.

Converts easily to and from binary

Binary	Hex	Decimal
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	A	10
1011	B	11
1100	C	12
1101	D	13
1110	E	14
1111	F	15

Binary, Hexadecimal, Decimal

Each four bits map to a hex digit.

Hexadecimal prefix 0x????

– No inherent value, just means "treat as a hex number"

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0x94D3
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Binary	Hex	Decimal
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	A	10
1011	B	11
1100	C	12
1101	D	13
1110	E	14
1111	F	15

Hexadecimal to Decimal

Convert each hex digit into 4 bits.
Convert binary to decimal.

Example:

0x94D3

= 1001 0100 1101 0011

$= 2^{15} + 2^{12} + 2^{10} + 2^{7} + 2^{6} + 2^{4} + 2^{1} + 2^{0}$

= 32768 + 4096 + 1024 + 128 + 64 + 16 + 2 + 1

= 38099 (decimal)

Conversions Between Number Systems

Try some!

http://www.mathsisfun.com/binary-decimal-hexadecimal-converter.html

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Take-Away Points

- Symbols represent values
- Number systems
- Binary
- Hexadecimal

- When do computers use decimal, octal, and hexadecimal numbers?

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