

# CS140:

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# !!! MATH WARNING !!!

TODAY'S LECTURE CONTAINS TRACE  
AMOUNTS OF  
ARITHMETIC AND ALGEBRA

# Overview/Questions

- What gives a number its value?
- What is a number system?
- I've heard that computers use binary numbers. What's a binary number?
- What kind of numbers do computers store and manipulate?

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## Background: Numbers

### **Natural Numbers**

Zero and any number obtained by repeatedly adding one to it.

Examples: 100, 0, 45645, 32

### **Negative Numbers**

A value less than 0, with a – sign

Examples: -24, -1, -45645, -32

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# Numbers

## **Integers**

A natural number, a negative number, zero

Examples: 249, 0, -45645, -32

## **Rational Numbers**

An integer or the quotient of two integers

Examples: -249, -1, 0,  $\frac{3}{7}$ ,  $-\frac{2}{5}$

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# Numbering Systems

A numbering system assigns meaning to the position of the numeric symbols.

For example, consider this set of symbols:

642

What number is it? Why?

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# Numbering Systems

It depends on the numbering system.

642 is  $600 + 40 + 2$  in **BASE 10**

The **base** of a number determines the number of digits (e.g. symbols) and the value of digit positions

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## Positional Notation

Continuing with our example...

642 in base 10 *positional notation* is:

$$\begin{array}{rclcl} 6 \times 10^2 & = & 6 \times 100 & = & 600 \\ + 4 \times 10^1 & = & 4 \times 10 & = & 40 \\ + 2 \times 10^0 & = & 2 \times 1 & = & 2 \end{array} = 642 \text{ in base 10}$$

This number is in  
base 10

The power indicates  
the position of  
the number

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# Positional Notation

$$642 \text{ is } 6_3 * 10^2 + 4_2 * 10^1 + 2_1 * 10^0$$

B is the base  
of the number

As a general form:

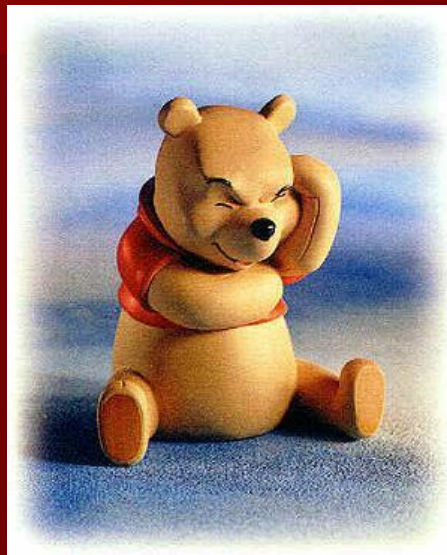
$$d_n * B^{n-1} + d_{n-1} * B^{n-2} + \dots + d_1 * B^0$$

n is the number of  
digits in the number

d is the digit in the  
i<sup>th</sup> position  
in the number

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## What Would Pooh Do?



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# Binary Numbers

Digital computers are made up of electronic circuits, which have exactly 2 states: **on and off**.

Computers use a numbering system which has exactly **2 symbols**, representing on and off.

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# Binary Numbers

Decimal is base 10 and has 10 digits:

**0,1,2,3,4,5,6,7,8,9**

Binary is base 2 and has 2, so we use only 2 symbols:

**0,1**

For a given base, valid numbers will only contain the digits in that base, which range from 0 up to (but not including) the base.

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# Counting...

Let's remember Kindergarten...

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## Binary Numbers and Computers

A **binary digit** or **bit** can take on only these two values.

Low Voltage = 0

High Voltage = 1

all bits have 0 or 1

Binary numbers are built by concatenating a string of bits together.

**Example:** 10101010

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## Positional Notation: Binary Numbers

Recall this general form:

$$d_n * B^{n-1} + d_{n-1} * B^{n-2} + \dots + d_1 * B^0$$

The same can be applied to base-2 numbers:

$$1011(\text{binary}) = 1 * 2^3 \quad (1 * 8)$$

$$+ 0 * 2^2 \quad (0 * 4)$$

$$+ 1 * 2^1 \quad (1 * 2)$$

$$+ 1 * 2^0 \quad (1 * 1)$$

$$1011(\text{binary}) = 8 + 0 + 2 + 1 = 11(\text{decimal})$$

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## Converting Binary to Decimal

***What is the decimal equivalent of the binary number 01101110?***

*(you try it! Work left-to-right)*

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# Converting Binary to Decimal

***What is the decimal equivalent of the binary number 01101110?***

$$\begin{aligned}0 \times 2^7 &= 0 \times 128 = 0 \\+ 1 \times 2^6 &= 1 \times 64 = 64 \\+ 1 \times 2^5 &= 1 \times 32 = 32 \\+ 0 \times 2^4 &= 0 \times 16 = 0 \\+ 1 \times 2^3 &= 1 \times 8 = 8 \\+ 1 \times 2^2 &= 1 \times 4 = 4 \\+ 1 \times 2^1 &= 1 \times 2 = 2 \\+ 0 \times 2^0 &= 0 \times 1 = 0 \\&= 110 \text{ (decimal)}\end{aligned}$$

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# Converting Binary to Decimal

***Try another one. What is the decimal equivalent of the binary number 10101010?***

*(you try it! Work left-to-right)*

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# Converting Binary to Decimal

***Try another one. What is the decimal equivalent of the binary number 10101010?***

$$\begin{aligned}1 \times 2^7 &= 1 \times 128 = 128 \\+ 0 \times 2^6 &= 0 \times 64 = 0 \\+ 1 \times 2^5 &= 1 \times 32 = 32 \\+ 0 \times 2^4 &= 0 \times 16 = 0 \\+ 1 \times 2^3 &= 1 \times 8 = 8 \\+ 0 \times 2^2 &= 0 \times 4 = 0 \\+ 1 \times 2^1 &= 1 \times 2 = 2 \\+ 0 \times 2^0 &= 0 \times 1 = 0 \\&= 170 \text{ (decimal)}\end{aligned}$$

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# Converting from Decimal to Other Bases

**Algorithm (process) for converting number in base 10 to other bases**

While (the quotient is not zero)  
Divide the decimal number by the new base\*  
Make the remainder the next digit to the left in the answer  
Replace the original decimal number with the quotient

\* Using whole number (integer) division only.  
Example:  $3 / 2$  gives us a quotient of 1 and a remainder 1

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## Converting Decimal to Binary

What is the binary equivalent of the decimal number 103?

$103 / 2 = 51$ , remainder 1 → rightmost bit

$51 / 2 = 25$ , remainder 1

$25 / 2 = 12$ , remainder 1

$12 / 2 = 6$ , remainder 0

$6 / 2 = 3$ , remainder 0

$3 / 2 = 1$ , remainder 1

$1 / 2 = 0$ , remainder 1 → leftmost bit

103 (decimal) = 1 1 0 0 1 1 1 (binary)

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## Converting Decimal to Binary

Now you try one. What is the binary equivalent of the decimal number 201?

*Recall the algorithm:*

While (the quotient is not zero)

    Divide the decimal number by the new base\*

    Make the remainder the next digit to the left in the answer

    Replace the original decimal number with the quotient

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# Converting Decimal to Binary

What is the binary equivalent of the decimal number 201?

$201 / 2 = 100$ , remainder 1 → rightmost bit

$100 / 2 = 50$ , remainder 0

$50 / 2 = 25$ , remainder 0

$25 / 2 = 12$ , remainder 1

$12 / 2 = 6$ , remainder 0

$6 / 2 = 3$ , remainder 0

$3 / 2 = 1$ , remainder 1

$1 / 2 = 0$ , remainder 1 → leftmost bit

201 (decimal) = 1 1 0 0 1 0 0 1 (binary)

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# Binary and Computers

## Byte

8 bits – a common unit of computer memory.

## Word

A computer **word** is a group of bits which are passed around together during computation.

The **word length** of the computer's processor is how many bits are grouped together.

- 8-bit machine (e.g. Nintendo Gameboy, 1989)
- 16-bit machine (e.g. Sega Genesis, 1989)
- 32-bit machines (e.g. Sony PlayStation, 1994)
- 64-bit machines (e.g. Nintendo 64, 1996)

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# Common Number Systems

Binary – base 2, has 2 symbols:

0, 1

Octal – base 8, has 8 symbols:

0, 1, 2, 3, 4, 5, 6, 7

Decimal – base 10, has 10 symbols:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Hexadecimal - base 16 has 16 digits:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

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## Why Hexadecimal?

Base 16 is a multiple of  
Base 2:

$$16 = 2^4$$

Each four bits  
map to a hex digit.

Converts easily to and  
from binary

Binary	Hex	Decimal
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	A	10
1011	B	11
1100	C	12
1101	D	13
1110	E	14
1111	F	15

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# Binary, Hexadecimal, Decimal

Each four bits  
map to a hex  
digit.

Hexadecimal prefix  
0x????

- No inherent value,  
just means “treat as  
a hex number”

0x94D3

Binary	Hex	Decimal
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	A	10
1011	B	11
1100	C	12
1101	D	13
1110	E	14
1111	F	15

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## Hexadecimal to Decimal

Convert each hex digit into 4 bits.

Convert binary to decimal.

Example:

0x94D3

= 1001 0100 1101 0011

=  $2^{15} + 2^{12} + 2^{10} + 2^7 + 2^6 + 2^4 + 2^1 + 2^0$

=  $32768 + 4096 + 1024 + 128 + 64 + 16 + 2 + 1$

= 38099 (decimal)

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# Conversions Between Number Systems

***Try some!***

<http://www.mathsisfun.com/binary-decimal-hexadecimal-converter.html>

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## Take-Away Points

- Symbols represent values
  - Number systems
  - Binary
  - Hexadecimal
- 
- When do computers use decimal, octal, and hexadecimal numbers?

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