CS201: Computer Vision Tracking: Kalman Filter and Multi-Target Tracking

John Magee
21 November 2014

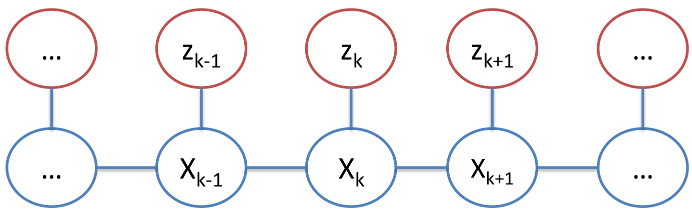
Slides Courtesy of
Diane H. Theriault (deht@bu.edu)

Question of the Day:

- How can we use measurements to estimate state?
- Kalman Filter: http://www.cs.unc.edu/~welch/media/pdf/kalman_intro.pdf

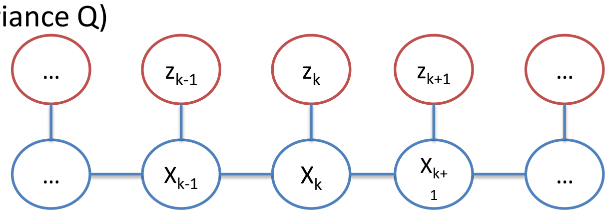
State vs Measurements

- Graphical model
- Goal: Estimate state by using measurements



State Changes Over Time

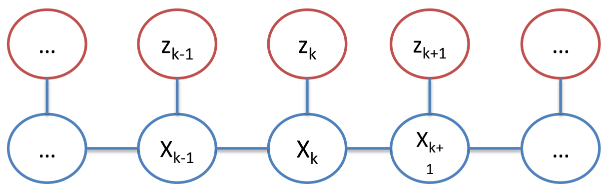
- Markov assumption: next state depends only on immediately preceding state
- State evolution function: A: $x_{t+1} = A(x_t)$
- Corrupted by noise (to absorb what you can't model)
  $x_{t+1} = A(x_t) + w_t$ (w is noise term. ex: Gaussian with covariance Q)



- Example: state = [pos vel]. A = constant velocity

Measuring State

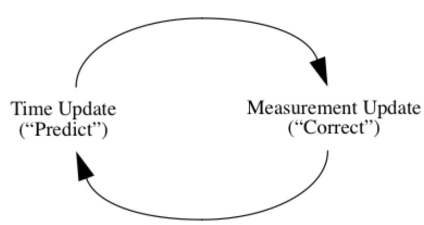
- Measurement Model: H
- Measurements: $z_t = H(x_t)$
- Corrupted by noise $z_t = H(x_t) + v_t$
  (v is noise term. ex: Gaussian with covariance R != Q)



- Ex: state is [pos vel]. Measurement is just pos

Kalman Filter Concept

- Predict / Correct
- Each state estimate has associated uncertainty (P_k)
- Recursive: each estimate uses previous estimate (not just data)



Kalman Filter Prediction

- Given:
  - Previous state estimate (x_{k-1})
  - Previous uncertainty (P_{k-1})
  - Linear State evolution fn as matrix A
  - Process noise (Q)
- Output:
  - Prediction (guess/estimate) of state at next time (x^-_k)
  - Certainty of that estimate (P^-_k)
  - (Predicted measurement: H x^-_k)

$$\hat{x}^-_k = A\hat{x}_{k-1} + Bu_{k-1}$$

$$P^-_k = AP_{k-1}A^T + Q$$

Kalman Filter Update

- Given:
  - Current state prediction (x^-_k) and uncertainty (P^-_k)
  - Measurement prediction (H x^-_k)
  - Actual measurement (z_k)
  - Measurement noise (R)
- Output:
  - Updated state estimate x_k

$$\hat{x}_k = \hat{x}^-_k + K_k(z_k - H\hat{x}^-_k)$$

$$K_k = P^-_k H^T (HP^-_k H^T + R)^{-1}$$

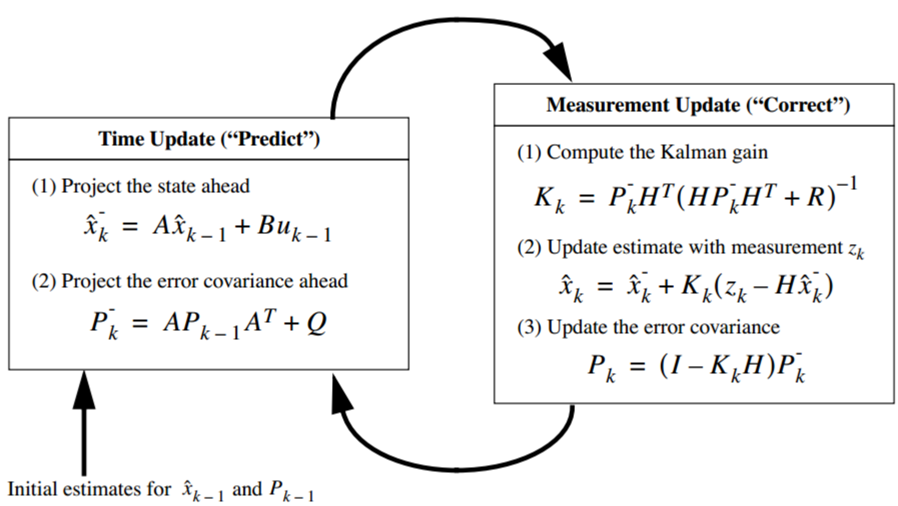
$$P_k = (I - K_k H)P^-_k$$

Kalman Filter Update

- Notes:
  - "Innovation" (z_k - H x^-_k) – the difference between the measurement prediction and observation
  - State update is basically a weighted average with a special weight K, the "Kalman Gain"

$$\hat{x}_k = \hat{x}^-_k + K_k(z_k - H\hat{x}^-_k)$$
$$P^-_k = AP_{k-1}A^T + Q$$
$$K_k = P^-_kH^T(HP^-_kH^T + R)^{-1}$$

  - Kalman gain both:
    - Transforms from the measurement space to state space
    - Balances the process noise and measurement noise



Noise Terms

- Process Noise (Q): how much uncertainty do you expect in your state evolution?
  - Ex: bats fly 10m/s. frame rate 131.5 fps :
    7 cm per frame.
- Measurement Noise: how much uncertainty do you have in your measurements?
  - Ex: with three cameras, we can use camera geometry to estimate our expected uncertainty

Kalman Filter for Smoothing

- That's easy: For every time step, use the state estimate instead of the state backed out from the measurement

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-)$$

Kalman Filter for Scoring

- After prediction: you have a probability distribution: mean and covariance of a Gaussian centered on predicted future measurement

$$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1}$$

$$P_k^- = AP_{k-1}A^T + Q$$

$$H\hat{x}_k^-$$

$$(HP_k^-H^T + R)$$

- Plug in the observed measurement to get probability
- Take product (sum logs) over all measurements

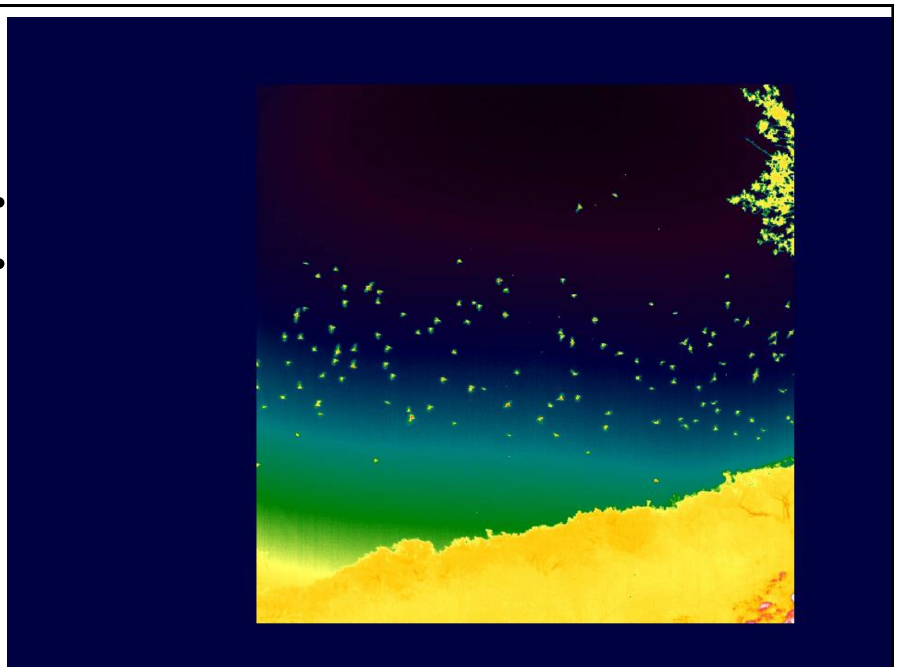
Discussion Questions:

- What does the Kalman filter give you?
- What do you need to know to use the Kalman filter?

Question of the Day (2):

- How can we simultaneously track many objects?

•
•



Multi-target tracking

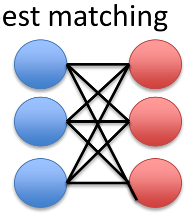
- Many detections in each frame
- Need to put them together somehow

- Two problems:
  – State estimation (e.g. Kalman Filter)
  – **Data Association**

  – Data Association != Kalman Filter

Assignment Problem

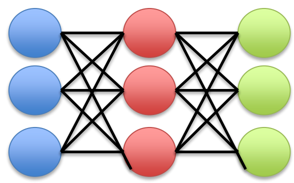
- Ex: Match workers and jobs
- Ex: Measurements from two consecutive frames
- Bipartite graph. Weights on each edge
  Find lowest cost / best matching



- Hungarian / Kuhn-Munkres assignment algorithm
- POLYNOMIAL (don't let anyone tell you otherwise)
- http://en.wikipedia.org/wiki/Hungarian_algorithm

Multi-dimensional Assignment

- Tri-partite (or higher) matching
- Ex: measurements from 3 or more frames of video.



- This is NP Complete (but we won't let that stop us)
  http://www.sce.carleton.ca/faculty/chinneck/po/Chapter12.pdf
  http://www.sce.carleton.ca/faculty/chinneck/po/Chapter13.pdf

Multiple Hypothesis Tracking

- "Choose the best possible set of tracks so that each measurement is used only once"
- How to formulate data association task mathematically?
- Kalman filter gives us state estimation, but we have to put the measurements together in the right order

Multiple Hypothesis Tracking

- Build out the MHT trees
- Set up corresponding matrices
- Score each leaf (using a Kalman filter)

- Formulate Integer Programming problem.

Practical Strategies

- Windowing
- Gating

Practical Considerations

- Missing measurements and new tracks (dummy measurements)
- Coasting